Chapter 6

THE ECONOMICS OF LEVERAGED LEASING

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§ 6:1 Introduction

A leveraged lease generally involves the acquisition of an item of capital equipment for a period equal to most, but not all, of the equipment's anticipated economic life and the sale of the residual value and the tax benefits of ownership to another party in exchange for a lease rate that is lower than the debt rate that would have applied to a purchase of the equipment. Although a leveraged lease is a rather complex form of financing with documents that are measured in inches rather than pages, its particular economics arise from only three of its features: (a) the involvement of three parties: a lessor, a lessee, and a lender who provides (b) *non-recourse* debt at a (c) substantial degree of leverage. Firstly we will consider leveraged leasing from the point of view of the lessor (owner) to find out why it chooses to own equipment that will spend most of its life in someone else's employ, how it analyzes its return, and how it chooses the leverage and debt amortization schedule. Next, we will look at the transaction from the lessee's viewpoint to understand why the lessee chooses to rent, rather than own, its equipment. Finally, we will look at lessee lease-buy analysis.

§ 6:2 The Classic United States Leveraged Lease

Let us consider a fifteen and one-quarter year lease of new equipment with a tax life of seven years. Let us assume the equipment costs $1 million, and the lessor wishes to purchase it with $200,000 of its own cash and $800,000 borrowed from lenders at 7.5 percent interest. The lessor assumes that it sells the equipment for $200,000 (the residual value). The lessee has an early buyout option (an "EBO") after ten and one-quarter years. The lessor pays a fee of $5,000 to the broker. Immediately upon purchase, the lessor will lease the equipment to the lessee for fifteen

and one-quarter years. Rents are paid on the same day the debt services are due, and the rents always are sufficient to pay debt services. The rents and debt services have been calculated using optimization techniques (see below).

The most important feature of the loan to the lessor is its non-recourse nature. That is, the lender agrees that the lessor is not personally liable for repayment of the loan and that the lender will look only to the rent received from the lessee for repayment. To secure this type of loan, the lessor must assign its interest in the lease to the lender. Rent and other payments made under the lease go directly to the lender (or its agent) and debt services due are deducted before the difference (if any) is sent to the lessor. The lessor also must give a first mortgage of the equipment to the lender.

The effect is substantially to remove the credit of the lessor from the transaction. The lessor will not record the loan as a liability or the equipment as an asset for financial reporting purposes. However, the lessor will report the cash investment as an asset and the deferred taxes as a liability.

Because the lessor, not the lessee, has borrowed the money on a non-recourse basis, the lender must look to the credit of the lessee for repayment of its loan. Thus, if rents are not paid and therefore the loans are in default, the lender may claim damages only against the lessee through the lease because the obligation to pay rent is in the lease. However, because the lender does not sign the lease, its claim depends on the lessor's assignment of its rights under the lease to the lender. The lender may also claim title to the specific equipment subject to the lease. Since the lender will have lent no more than 80 percent of the equipment's cost, it has a favorable collateral ratio. As a practical matter, in the event of a lessee bankruptcy, the trustees have often elected to pay rentals rather than risk the loss of the equipment.

§ 6:2.1 Before-Tax Analysis

The first step in the economic analysis of this lease is to calculate the amount of the lessor's investment. The lessor has invested $200,000 in cash, has paid a broker's fee of $5,000 and has borrowed $800,000 on a non-recourse basis. Is its investment $205,000 or $1,005,000? In this regard, the importance of the non-recourse nature of the debt cannot be over emphasized. Had the lessor borrowed the $800,000 from its bank and bought the equipment, all the cash flows would be the same, including the cash flows arising from the tax aspects of ownership, but then the lessor's investment would be the purchase price of the equipment plus fee, the lease would not be a leveraged lease, and the economic analysis would give drastically different results. If an ordinary leveraged lease were analyzed in terms of recourse loans, the economic yield to the lessor would be minuscule. In our example, the lessor's investment is $205,000, which equals the equipment cost plus broker's fee less the amount of non-recourse debt.

We now have the lessor's investment of $205,000 and the before-tax cash flows from the lease consisting of the excess of rent over debt services of $122,308 and the residual value of $200,000 (see §6:9.2). By ordinary present worth techniques, we can calculate the yield to be 2.573 percent per year. The "multiple investment sinking fund" method of analysis discussed below would provide the same result. This rate of return is lower than the debt interest rate, indicating

an inferior rate of return for the lessor. Lessors generally require a return rate higher than the debt rate to compensate for the greater risk arising from the lessor's secondary credit position (the lenders have the first mortgage) and the uncertainty that the residual value will be as large as assumed. In the foregoing example, the yield would be zero with a residual value of $82,692 and the lessor would suffer a loss with a lower residual value.

The foregoing result demonstrates that a leveraged lease provides an unsatisfactory rate of return when analyzed without including the tax aspects of equipment ownership. This result arises from the fact that lease rates on the leveraged leases are lower than debt rates on the loan. From the point of view of the lessee, this is the main motivation to lease, rather than purchase, the asset in the first place. Accordingly, we will now discuss the modifications to the lessor's normal tax payments that occur as a result of the lease.

§ 6:2.2 Tax Benefits

Current tax law in the United States subsidizes the purchase of new capital equipment for tax paying companies. Since 1980, we have seen large and rapid changes in the amount of tax benefits available. The Economic Recovery Tax Act of 1981 introduced the Accelerated Cost Recovery System ("ACRS"). Under this system, assets had a tax life of 3, 5, 10 or 15 years. Most leased equipment had a tax life of 5 years. An Investment Tax Credit ("ITC") was available, and the purchaser of new equipment was entitled to deduct 10 percent of the equipment price from its taxes for the year in which the equipment was placed in service. The Tax Equity and Fiscal Responsibility Act of 1982 reduced the ITC benefit available. The Tax Reform Act of 1986 (the "1986 Act") repealed the ITC except for a limited amount of transitional property. Depreciation schedules were changed and two new tax lives were added. Tax rates were lowered from 46% to 34%, and many base broadening proposals were introduced. Corporate tax rates are currently (2007) 35%. The Job Creation and Worker Assistance Act of 2002 allowed an additional thirty percent of the equipment price to be depreciated in the first year. The Jobs and Growth Tax Relief Reconciliation Act of 2003 changed this bonus depreciation to 50% for qualified equipment placed in service between May 6, 2003 and December 31, 2004.

Our example will be calculated using the current tax rules. The meaning of the economic analysis does not change when other tax assumptions are used, although the exact tax numbers do change.

The only real tax benefit available to purchasers of new equipment under the 1986 Act is an accelerated depreciation deduction. This confers a benefit provided that the lease term for the equipment is longer than its depreciable life, because substantial taxes would then be deferred. For our lease, the deduction available the first year is $571,429 and the deduction available the second year is $122,449. Our example assumes a basis of $1,000,000 and a seven-year tax life. These depreciation deductions are of value to the lessor only if it has income from other sources in excess of these deductions and if the lessor is not subject to the Alternative Minimum Tax. If this is the case, the lessor will pay $205,279 less in taxes the first year and $32,744 less in taxes the second year (at a 35% rate). The amount of taxes saved declines thereafter.

The timing of the tax savings is the relevant benefit in leasing, not the change in total tax liability. The broker's fee is amortized on a straight-line basis over the life of the lease. The other tax benefit available from our lease is the deduction for interest on the loan, which amounts to $15,000 in the first year. The rents paid by the lessee are the fully taxable income of the lessor. The taxable income of the lessor for each year of the lease is shown in §6:9.1. Under the assumption that the lessor pays substantial taxes each year (including future years), the negative tax payments in the last column are treated as cash inflows. Note the general pattern: taxes are reduced in the first years of the lease and are paid in later years. This pattern is a general feature of leveraged leases, and the benefit will show up in any sort of present value analysis. The tax payments are shown in an annual table. The actual date on which the payments are made depends on the lessor's tax payment method. For a calendar-year corporation, payments of estimated tax are made on the fifteenth day of April, June, September, and December. Any additional tax due is paid on the fifteenth day of March of the following year. Penalties are prescribed for insufficient payments of estimated tax. The table assumes that the lessor pays 100% of its tax liability on a current basis. The yield obtained may be very sensitive to the exact amount of taxes saved on each tax payment date, especially during the first year. This in turn depends on the tax planning and management of the lessor.

§ 6:2.3 After-Tax Yield

We now have all the information needed to calculate the lessor's yield on its investment, namely the amount of its investment and the cash flow due to the investment. The cash flow is shown in §6:9.2 by year and is equal to the rent plus the residual value less the debt service less the taxes.

The first point of interest in the above cash flow is the rapid return of the investment. Because of the 50% bonus depreciation credit, the initial tax loss gives a credit that exceeds the initial investment. Thereafter there is a small effective re-investment caused by a 3-month interim period payment (discussed below) that is returned by the fifth year and, in fact, cash over and above this amount is paid to the lessor through 2014. Thereafter, the payment of taxes reduces the lessor's cash position until it finally remains with a profit of $76,250 at the end of the lease term.

The usual method used for obtaining the yield on a leveraged lease is the "multiple investment sinking fund" or "MISF" method. This method is very similar to the usual internal rate of return method, but it uses one rate (the yield rate) for the investment stage and another rate (the sinking fund rate) for the sinking fund stage (the period when the lessor has received cash exceeding the initial investment). The MISF method provides the same result as the internal rate of return method when there is only one rate used for the two stages in a transaction. Therefore, the MISF method is a generalization of the internal rate of return method rather than a completely new and different method of analysis.

The yield found from the MISF method is the yield on the lessor's investment while the lessor has an outstanding investment (as opposed to a sinking fund). That is, the yield rate when multiplied by the investment outstanding from time to time will equal the lessor's earnings. If, during the lease, the lessor's investment is repaid, and if a sinking fund develops due to the further receipt of cash, the investment balance will be zero and the sinking fund will earn interest

at a specified rate, called the sinking fund rate. The sinking fund earnings are compounded back into the sinking fund. When the cash flow eventually becomes negative due to tax payments, these payments will be made from the sinking fund until its balance is reduced to zero. If any further negative cash flows remain, they will be met by further investments by the lessor and the investment balance will increase and accrue earnings at the yield rate. The yield rate is the rate at which both the investment balance and sinking fund balance are zero at the end of the lease. In practice, there may be several investment stages and several sinking fund stages in a single lease analysis depending upon the details of the cash flows.

The calculation for our example is shown in §6:9.3 on a monthly basis for the first two years of the lease and for the last year of the lease. The investment in the first month is $205,000. The earnings of $1,196 in the first month are derived from the investment balance of $205,000 multiplied by the after-tax yield of 7.00% divided by twelve months. Subsequent earnings are calculated in the same way until a sinking fund is established at the end of the lease in 2023. At that point, the cash flow exceeds the sum of the earnings and the investment balance for the month in which the sinking fund occurs. The excess cash is used to establish a sinking fund. The sinking fund earnings are the sinking fund rate multiplied by the sinking fund balance. These calculations are performed for every month of the lease. §6:9.4 shows the annual totals for the full term. The sinking fund rate for this example is zero.

A yield for the forgoing cash flow and investment may be found by the usual "internal rate of return" method that is based on present value. However, the resulting yield would not be accepted as an accurate yield in the marketplace and would be higher than, or equal to, the yield found from the MISF method of analysis. The problem is the correct treatment of the lessor's position when it holds funds in excess of its investment plus profit, that is, the correct treatment of the sinking fund. The usual internal rate of return method would evaluate the sinking fund at the same rate as it would value the investment. That is, during the time that the lessor has a negative cash position (the time immediately after the lessor makes its investment), the internal rate of return method would impute earnings to the investment at the internal rate (the yield rate to be found from the analysis). As the lessor receives cash the investment would be paid back (with earnings) until it would be reduced to zero. As more cash is received, a sinking fund develops, which would earn money (interest) *at the same rate*. Then, as cash flows become negative, the sinking fund would be depleted until finally, at the end of the lease term, both the sinking fund balance and investment balance would be zero. However, implicit in this analysis is the idea that the lessor can earn a return at the yield rate on surplus funds available for the period of the sinking fund. The lessor would view these funds as only worth approximately the rates on its bank lines, or perhaps the rates on short-term debt instruments, or typically 3 percent per year after tax. Therefore, the lessor would reject the internal rate of return yield as too high. Another objection to the normal internal rate of return analysis as applied to leveraged leases is that mathematically it may give rise to more than one yield. However, this objection should be viewed as the mathematics trying to tell the analyst that something is wrong rather than as a basic fault with the method. In any case, our example deal has only a small sinking fund.

Please note that in this yield method the sinking fund is not established to "take care of tax payments." For purposes of analysis, tax payments may be met both by the sinking fund and by further investments by the lessor. In fact, the lessor may actually spend its "sinking fund" as it

receives it, and then meet tax liabilities as they come due with its own funds. The division into precise sinking fund and investment stages is a mathematical procedure used to find a yield rate, and the lessor may not have an actual sinking fund. However, the lease may temporarily give the lessor cash in excess of its investment plus inherent profit and the lessor may invest these funds while it has them. Therefore, the concepts of a "sinking fund" and "sinking fund rate" are pertinent, even if the exact sinking fund flows used in finding the yield are never achieved or even attempted. The yield derived from the multiple investment sinking fund method has certain desirable mathematical and logical features that recommend its use and the leveraged lease marketplace regards this yield as one of the pertinent measures of lessor return. Statement of Financial Accounting Standards No. 13 ("SFAS No. 13") specifies the multiple investment sinking fund method as the recommended method for recording book earnings with the stipulation that a zero sinking fund rate must be used.

Apparently, the yield the lessor receives from the lease depends upon the sinking fund rate chosen for the analysis, and ultimately the real yield obtained depends upon the lessor's ability to utilize the surplus funds that become available as a result of the lease. In the past, lessors typically evaluated a lease at a sinking fund rate of zero, and again at a rate of 3 percent per year after tax. Currently, the use of a zero sinking fund rate is nearly universal due to four main reasons. First is the requirement of a zero sinking fund rate for reporting book earnings. Second is the loss of the Investment Tax Credit, which delays the repayment of the lessor's investment from tax benefits. Third is the larger residual value assumptions that result in a substantial lessor investment at the end of the lease term and thus less time in the middle of the lease term for a sinking fund to develop. Fourth is the use of optimization methods, which seek to adjust rent and debt service to maximize economic benefit. Because the sinking fund rate will always be less than the yield rate and usually less than the debt rate, this has the effect of devoting cash flows to the lessor's investment and debt balance and thus minimizing the amount of the relatively less valuable sinking fund.

§ 6:2.4 Before-Tax Yield

Because the yield on an investment in a leveraged lease is calculated in terms of the after-tax cash flows, the "before-tax yield" is a misnomer. As discussed above, the actual yield calculated on the before-tax cash flows is a very low number. In the mid-1970's the after-tax yield described above was used as the "yield" and no before-tax yield was used. At that time, after-tax yields were about 20% per year. When the market became more competitive around 1980, it became common to express the after-tax yield as a before-tax "equivalent" yield by dividing it by one minus the tax rate. That is, at a 46% tax rate, a 10% after-tax yield is "equivalent" to an 18.5% before-tax yield. At the same time the yields fell to half their former levels. The new "before-tax" yields were then about 20%. Lessors must often compete for funds with others within their organizations. It is easier to use a before-tax yield, which is more similar to the usual interest rate on a loan, rather than to try to explain the method of calculating the after-tax yield. In 1986, Congress changed the tax rates, and lessors were confronted with tax rates, which varied from 46% to 34% over the term of the deal. The "gross up" procedure no longer worked, but a new "true" before-tax MISF yield was invented which solved this problem. The solution was to

perform the same analysis as the after-tax MISF yield described above, but instead of using an after-tax yield, the before-tax yield multiplied by the (variable) tax rate is used. Currently, most lessors seem to be using after-tax yields again.

§ 6:2.5 Effective Yield

A few lessors restate the after tax yield as an "effective" annual yield. In other words, if $1.00 in a savings account earning 10% (nominal annual) interest compounded monthly grows to $1.1047 in one year, the effective annual yield is 10.47% per year. Although this calculation may make sense for savings accounts, it makes no sense for leveraged leases. The investment balance in a leveraged lease changes throughout each year. Most computer programs calculate the yield on a monthly basis. The resulting yield is multiplied by twelve to produce the "nominal" yield. To convert this number to a higher annual effective yield adds nothing to the calculation. Most computer programs can also calculate the yield on a daily basis. This yield is converted to an effective monthly yield to make it more similar to the usual monthly yield.

§ 6:2.6 Book Earnings

In November 1976, the Financial Accounting Standards Board published SFAS No. 13, which addresses leveraged lease accounting. Since then, the lessor's earnings in a leveraged lease are booked in the same way that the lessor analyzes his yield: earnings are obtained by applying a constant yield rate to its outstanding investment on an after-tax basis. An estimated residual value may be included, but the sinking fund rate used must be zero. The effect of this is to conform the accounting treatment to the economic analysis. In practice, a large part of the lessor's earnings are recognized over the first few years of the lease term.

§ 6:2.7 Sinking Fund Risk

Our example has only a small sinking fund, and consequently the yield is not too sensitive to the sinking fund rate used. The after-tax yield changes from 7.000% to 7.062% when the sinking fund rate is increased from zero to 3.0% after tax. In the past, leveraged leases often had larger sinking funds, and lessors often spoke of a "sinking fund risk"; the risk that the lessor would not in fact be able to realize earnings on the sinking fund (surplus cash position) at the rate it originally assumed when it signed the deal. This risk prompted lessors to evaluate leases on the basis of a sinking fund rate of zero even when the short-term debt rates were historically high. The resultant yield was safer (and lower), and the lessor could view the sinking fund earnings as a potential upside factor, rather than assuming their existence at the beginning of the lease term.

§ 6:2.8 Residual Value Risk

The lessor owns the equipment at the expiration of a true lease. The sale price of the equipment at that time is called the residual value. In our example, we assumed that this value was $200,000. In the mid-1970's, lessors were inclined to bid deals without reference to the residual value and to think of the residual as a potential upside factor. However, lessors often included an estimate of the residual value when booking earnings. Over the years, the leveraged leasing

market became much more competitive and the rentals were lowered in recognition of a more reasonable residual value estimate. Many items of equipment have now come off lease, and lessors are developing a track record in residual values.

In 1986, changes to the tax law repealed the Investment Tax Credit and lengthened depreciable lives. The amount of tax benefits in a lease therefore decreased. Consequently, to achieve the same yield, the lessor must increase the rent. In order to preserve business, lessors now use a larger and more carefully estimated residual value. By assuming larger residual values the lessor can raise the rent by a smaller amount and still achieve the same yield even with the reduced amount of tax benefits. Inflation can have a substantial effect on residual value. For a ten-year lease term, an average inflation rate of 7.18% per year will double the residual value. For a fifteen-year lease term, a rate of 4.73% per year will double the residual value. However, estimating inflation rates is difficult. The risk that the residual value will be less than assumed is now one of the greatest risks in a leveraged lease.

§ 6:2.9 Credit Risk

If the lessee ceases to pay rents, the non-recourse loans will be in default, and the lenders will foreclose on the leased equipment. The amount of money that the lessor would lose must be calculated on an after-tax basis, and would include taxes on income resulting from forgiveness of the lease debt. In our example, the amount of loss is 20.5% in the first year, the amount of the investment. In the last year it is 20%, the amount of the residual value. For intermediate years, the amount of loss is about 27%. The point is that the lessor's credit risk is substantial throughout the lease term. Even for leases with a smaller residual value, the risk amount is high through the middle of the term and only falls at the end.

Note that the risk amount is not the same as the investment balance shown in §6:9.4. The investment balance is the basis for the yield calculation. The lessor has a substantial risk position until the very end of the lease term, even though the investment has been returned through tax savings by 2009. This is true because the tax savings taken to reduce the investment balance (and raise the yield) are recaptured upon the conversion of the asset. All that really reduces the risk amount is cash from the lessee paid to the lessor (cash paid to the lenders does not help) and receipt of the residual value. This is *quite different* from a loan. In a loan, the risk amount and the investment balance would be identical, since the amount at risk and the investment both equal the loan principal outstanding. The lessor should be aware that, unlike a loan, the yield does not have any relationship to the risk position. The lessor cannot raise the rent to increase the yield a few points to compensate for risk, as it would if it were lending money. With a loan, the amount at risk is the principal outstanding and increasing the interest rate one percent adds additional income exactly proportional to the principal outstanding and thus to the risk amount. Therefore, the lender receives additional income in proportion to the risk. This simple relationship *does not hold true* for a leveraged lease. Because the risk amount is relatively constant over much of the lease term, it is more sensible for the lessor to add a constant amount to the rent in compensation for risk rather than to look at the yield.

We find that most lessors think that the yield (however measured) is about the same as the interest rate on a loan. That is, a leveraged lease with a yield of 7% per year after-tax is "like" (if

not equivalent to) a loan with the same interest rate. This view may be tempered with considerations such as the different risk posture and the extent to which the yield depends upon the realized residual value.

§ 6:2.10 Tax Rate Change Risk

Because some of the economic benefit of leveraged leasing comes from tax benefits, it is to be expected that changes in the tax law or rates can have a substantial economic impact on a transaction. Careful documentation can minimize the risk of tax law changes, since changes before commitment can give risk to unwinds or price adjustments, while changes after commitment are usually "grand-fathered."

Tax rate changes are a real risk of leveraged leasing, a point that was brought home when Congress changed the tax rate from 46% to 34% by the 1986 Act. The previous schedules in our example are all based on the current corporate tax rate of 35%. A tax rate change can be a benefit or a detriment to a leveraged lease depending upon when it occurs. An increase in the tax rate at the "cross-over" (the time when the lease stops showing losses and becomes taxable) will decrease the profit and yield because tax savings taken at a lower rate must be repaid at a higher rate. The opposite is true for a decrease in the tax rate at the crossover.

Leveraged leases vary in their sensitivity to tax rate changes, and generally the sensitivity is less the lower the leverage because interest deductions are lower and because the changes in cash flow are smaller relative to the investment. In some cases, state taxes can have effects similar to a tax rate change. If the lessor cannot consolidate the lease for state tax purposes, the tax losses will not do any good, while state tax payments still must be made in later years depending upon the state carry-forward rules.

Although lessors' concern has focused on adverse tax rate changes, the 1986 Act resulted in windfall profits for lessors with leases written in the sixties and seventies. These leases were often highly tax-intensive, and the tax rate decrease reduced the tax liabilities of these leases. Furthermore, paragraph 46 of SFAS No. 13 requires that all the gain (or loss) must be recognized "in the year in which the assumption is changed." That is, the lessor must use "catch up accounting," and book the gain (or loss) all at once.

Although the tax rate change risk (or reward) can be substantial for a single leveraged lease, the risk for a portfolio of leveraged leases can be much lower. If at the time of the tax rate change the lessor has some leases in the tax-loss phase and some in the tax-gain phase, the effect of the tax rate change will be decreased. Of course, if the lessor has mostly leases of the same type starting at the same time, this cancellation effect will not occur. The lessor can manage its portfolio to minimize tax rate change effects and also to achieve desirable overall cash flow characteristics.

§ 6:2.11 Tax Base Risk

The lessor also runs the risk that it will not continue to have net taxable income from other business activities in excess of the losses in the early years of the lease term. In this case, the tax

losses do not immediately give rise to tax savings, the lessor will be in a tax loss carry-forward situation, and the cash inflows we have assumed will be deferred. The yield will be decreased, and if the carry-forwards expire, the inherent profit will be lower. Clearly, the lessor should consider the effect of its leveraged leases in its overall tax planning.

§ 6:2.12 Alternative Minimum Tax Risk

The 1986 Act added an additional risk to leveraged leasing. The 1986 Act provided for a new type of minimum tax, which must be paid if it is higher than the regular tax. Basically, the excess of MACRS depreciation over 150% declining balance switching to straight-line depreciation in a leveraged lease is an item of tax preference. If the lessor has enough tax preferences from its leveraged leases and other business activities, the lessor becomes a minimum taxpayer and the cash flows from §6:9.1 and §6:9.2 will not occur.

§ 6:2.13 Termination Values

All leveraged leases provide for a lump sum payment to the lessor in the event of equipment destruction, lessee default, or other early termination of the lease. These different cases often provide for different payment amounts, the difference being the amount of built-in residual value that will be provided to the lessor. The termination value is first used to repay the non-recourse debt, and the excess is then the property of the lessor. Termination values are equal to the total of equipment cost plus fees at the start of the lease term. Because termination values protect the lessor's yield and because yield earnings are recognized on an accelerated basis, termination values remain at this level for several years and then gradually diminish until they equal the residual value at the end of the lease term. An example of termination values is shown in §6:9.6.

In practice, termination values are a negotiated item. Lessors may ask for a higher termination value for a voluntary termination than they do for a casualty loss. Lessees have argued that no residual value should be included in the case of a termination for obsolescence because the lessor should bear this risk; lessors have argued the opposite.

§ 6:2.14 Early Buyout Option

Our example lease includes an early buyout option ("EBO"), which permits the lessee to purchase the leased equipment on January 1, 2018 for $616,487. In our example lease, the lessor's yield if the EBO is exercised is the same as the full-term yield, namely 7% after tax, but the deal could have been structured for a different EBO yield. The EBO is similar to a termination value in that it generally protects the lessor's yield as well as repays the lenders. Our EBO is a "deferred EBO", which means that the EBO is paid in five installments, one on January 1, 2018 and four more which match the lessor's tax liability in 2018. The deferred EBO is used because it results in a lower present worth of the EBO to the lessee. It does not lower the lessor's yield because the lessor will be in a sinking fund state, and the sinking fund rate is zero.

The EBO is a structuring technique invented during the 1990's, and prior to that time lessors claimed that an EBO was contrary to IRS guidelines (see below) because it appears to give the lessee a bargain purchase option, and generally gives the lessee the right to the residual upside.

To satisfy IRS guidelines, the deal has been structured with an EBO "compulsion test" which states that the EBO must not be less than the present value of the remaining rents plus an assumed fair market value (higher than the 20% assumed for economic analysis). Therefore, the EBO does not represent a "bargain" purchase option.

§ 6:2.15 IRS Guidelines

The Internal Revenue Service (the "IRS") has published guidelines relating to the structuring of leveraged leases. By following the guidelines, which are contained in Revenue Procedures 75-21, 75- 28, 76-30, and 79-48, the parties may expect to receive a favorable ruling from the IRS to the effect that the transaction is a lease for federal income tax purposes. Today, it is uncommon for a lessor to seek a ruling, but the guidelines are followed for leveraged leases in the institutional market.

The first point made in the guidelines is that the lessor must have a minimum unconditional at-risk investment of at least 20% of the cost of the leased equipment at the beginning of the lease term. That is, the lessor must pay 20% of the cost of the leased equipment (or assume personal liability in the same amount) by the time the equipment is first placed in service and may not be entitled to a return of the investment by any member of the lessee group through any form of unwind. Further, this investment must be maintained throughout the lease term. This means that the lessor may not receive the bulk of the early rentals by deferring debt service and thereby receive the cash sooner.

The leased equipment must have a minimum remaining value at the end of the lease, and the lessor, not the lessee, must bear the risk and reward for this value. The lessor must represent and demonstrate that the leased equipment will have a value of at least 20% of its original cost (without regard for inflation) at the end of the lease term. In addition, the equipment must not be leased for more than 80% of its economic life. These representations must be net of any removal or conversion costs, must contemplate the use of the equipment by parties other than members of the lessee group, and must be supported by commercial feasibility studies when necessary.

The lessor may not have the right, nor presently intend to acquire the right, to require any party to purchase the leased equipment at other than its fair market value (except for performance guarantees by the manufacturer of the equipment). No member of the lessee group may have the right to purchase the equipment for less than its fair market value.

No member of the lessee group may pay for part of the cost of the equipment. This also applies to certain improvements during the lease term, but excludes maintenance. No member of the lessee group may lend to the lessor any of the funds necessary to acquire the equipment or guarantee any indebtedness created in connection with the acquisition of the equipment by the lessor.

The lessor must demonstrate that it expects to receive a profit on the lease apart from tax benefits. This requirement is satisfied if the sum of the rent and expected residual is greater than the sum of the debt service and investment. Also, the lessor must receive significant cash flow from the lease. This requirement is met if the total rent exceeds the total debt service by a

reasonable amount. This reasonable amount has been interpreted to be 2% of the investment multiplied by the number of years in the lease term.

If the rents in the lease are not level, the payments may give rise to prepaid or deferred rent. The IRS will not question unequal rent if the annual rent is either (a) always within 10% of the average of the rentals ("90-110" rents) or (b) within 10% of the average for at least the first two-thirds of the lease term, and for the remainder of the lease term, is no higher than the highest rent during the initial portion of the term and no lower than half the average rent during the initial portion. Currently, part (b) of the test is not used, and rent must satisfy part (a). The IRS may allow rents more uneven than the foregoing if there is a good business reason for the variation. In particular, rents that may vary in compensation for floating rates on the lease debt are allowed. Section 467 of the Internal Revenue Code of 1986, as amended (the "Code") may further limit the amount by which rent may increase. If Section 467 applies, then income from the rents must be recognized on a constant present value basis. Some lessors use level rents to avoid Section 467 problems. Others believe that the 10% test in the guidelines provides a safe harbor from Section 467 problems.

The uneven-rent test produces a serious structuring constraint. When the lessee evaluates the cost of the lease by measuring the present worth of the rents, the lessor may obtain a better bid by using a "low-high" rent structure. That is, rents are lower in the beginning of the lease term and higher at the end. Such a rent structure may show a present value cost lower than level rents for a given return to the lessor. However, the uneven-rent test constrains the amount by which the rents can vary.

Our example has been structured with "continuous" rents, meaning that a combination of advance and arrears rents is used in the same lease. Note the rather confusing rent payments in §6:9.5. The use of both advance and arrears rents creates more freedom to reduce the present value of rent while still passing the IRS uneven rent test.

As discussed in §6:4.1 below, the May 17, 1999 IRS final regulations allow that the timing for the payment of cash rents and the allocation of the rents for tax purposes can differ, subject to specific guidelines. The above discussion for the uneven-rent test applies to the taxable income allocation of the rents rather than the cash amounts received.

§ 6:3 Rent and Debt Structuring

Today's leveraged leasing marketplace is highly competitive. Lessees measure the cost of a lease by using the present worth of the rent. Large deals are won or lost on the basis of a few hundred dollars in present worth of rent per million dollars of equipment cost. Today, it is the universal practice to "optimize" a leveraged lease by using a computer program which chooses the amount of leverage, and the exact rent and debt service amounts in order to achieve the lowest present worth of rent. The best lease has the lowest possible present worth of rent, and achieves all the necessary constraints. This result can be obtained using a mathematical procedure known as "linear programming" (see below) as implemented in a computer program.

The variables to be found are the amounts of each rent and each debt service. The lessor usually wants to optimize for the lowest present worth of rent, either the full term rent or the rent for the EBO term plus the EBO amount. However, it is sometimes convenient to optimize for the lowest lease rate, or the highest yield or profit at fixed rents. Of course, the lowest present worth of rent is obtained at zero rent. The lessor must also enter requirements or "constraints" into the problem, such as, for example, the constraint that the after-tax yield must be at least 7%. There are usually many other constraints such as the IRS constraints. A representative list of constraints would consist of:

- The minimum yield (7% in our example).
- Debt service must not exceed rent.
- The IRS uneven-rent constraints (very important).
- The minimum total after-tax (book) profit.
- The minimum IRS profit of $1.00.
- The minimum IRS cash flow of 2% per year.
- The maintenance of minimum investment constraints each period.
- The EBO compulsion test (see §6:2.14 above).

Other constraints sometimes needed are:

- The minimum and maximum amount of debt.
- The minimum and maximum average life of the debt.
- Other rental shape constraints, such as level, high-low step, etc.
- Extra constraints on the free cash (rent less debt service).
- Specified value for the present worth of rent (for optimized yield).
- Specified values for certain rents or debt services.

These constraints must be chosen carefully. Only the necessary constraints must be put into the computer model so that the resulting structure will meet the lessor's requirements. If too many constraints are entered, it may turn out that a lower present worth of rent could have been obtained with fewer constraints.

For leases with an EBO, a common optimization goal is the "EBO Benefit to the Lessee". The EBO Benefit is the difference between the cost of the asset and the amount required to be put into a deposit at the start of the lease at a fixed rate of interest so that its value will appreciate sufficiently to cover both the rent payments and the EBO amount required. The optimization analysis then determines the rent and debt payments required to produce the lowest possible EBO Benefit while maintaining the required minimum MISF yield and satisfying all constraints specified in the lease structure.

§ 6:3.1 Linear Programming Models

Mathematical programming is a technique for solving constrained optimization problems. In these problems, the values of unknown quantities are subject to a series of constraints. A proposed set of values for the unknowns constitutes a “feasible solution” to the problem if, and only if, it satisfies all the constraints. In general, there may be many such feasible solutions. The aim is to find the “best” feasible solution. Depending on the specific application, the “best” feasible solution may be the solution that maximizes profit, minimizes cost, maximizes throughput, and so on. In leveraged leasing, the usual objective is to minimize the present worth of the rents. This goal is called the “objective function” and the feasible solution that produces the best possible value of the objective function is called the “optimal solution”.

The general problem described above is very difficult to solve analytically. If the constraints and objective function take a specific form, however, the problem can be classified as a “linear programming” problem and can be solved more readily. A relationship is “linear” when the effect of changing the value of one unknown quantity is directly proportional to the magnitude of the change. If, for example, a product is sold at a fixed price, then income will be related linearly to the number of units sold: each additional unit sold increases the income by the same amount. On the other hand, if volume discounts are offered, then the relationship is non-linear: the effect of increasing the number of units sold depends not only on the magnitude of the increase, but also on the number already sold. The leasing problem is a linear problem. For example, if a rent is increased by a given amount, the present value worth of that rent increases by an exactly proportionate amount.

A widely used “algorithm”, or method of solution, for linear programming problems is known as the “simplex” method. Computer implementations of this method are commercially available to handle problems with thousands of constraints. The method is an iterative one, which selects progressively “better” solutions until reaching a solution that is optimal. The number of iterations required to reach the optimal solution increases with the number of constraints in the problem. Theoretically, the number of iterations required increases quite rapidly; in practice, the simplex algorithm performs surprisingly well on real-world problems.

§ 6:3.2 Mathematical Framework for Linear Programming

(Note: The less mathematically inclined may wish to skip this section.) The standard form of the linear programming problem is expressed in terms of inequalities:

$$\text{minimize:} \quad c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$\begin{aligned}\text{subject to:} \quad & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n <= b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n <= b_2 \\ & \ldots \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n <= b_m \\ \text{with:} \quad & x_i >= 0 \quad \text{for all} \quad x_i\end{aligned}$$

where x_i are the unknowns and c_1, a_{ij}, and b_i are constant coefficients. An inequality can be equivalently expressed as an equation by adding a new variable called a "slack" which measures the degree to which the left-hand side of the inequality is less than the right-hand side:

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + S_i = b_i$$

If slack variables are added to each constraint, there are n+m unknowns (n "structural" and m "slack" variables) and m constraints. This yields (n + m) – m = n degrees of freedom. If we fix each of the slack variables to the corresponding right-hand-side value (b_1), we obtain a feasible solution with which to start the simplex algorithm. In this "all-slack" solution, the structural variables are all at zero level.

The all-slack solution is an example of a "basic" solution: a "basic set" of n variables are allowed to take on non-zero values, while the remaining "non-basic" variables have values of zero. If the constraint set is viewed as defining a convex space in n-dimensional space, basic solutions are located at the corners of the space.

For any such convex space and any linear objective function, the optimal solution will always lie at a corner (or possibly on the edge connecting two corners of equal value). The simplex method, accordingly, considers only basic solutions. At each iteration, it chooses a non-basic variable with promising marginal economics to bring into the basis and chooses a basic variable for it to replace. The effect is to move along an edge of the feasible space to an adjacent corner with better or equal objective value. When no basis change can improve the objective function, the simplex method has located the optimal solution.

If the starting basis contains artificial variables at non-zero levels or slack variables at negative levels, the basis is infeasible: the solution it represents falls outside the feasible region. The usual procedure in this case is to follow a two-phase simplex. In the first phase, a feasible solution is sought by minimizing the infeasibility rather than the true objective; in the second phase, the optimal solution is found.

It is possible for no solution to exist. For example, suppose that $x_1>3$ and $x_1<2$ are both part of the constraint set. In this case, no feasible solution can be found: the minimum infeasibility found in phase one will be positive. Infeasibility in a model is not always so obvious and, due to the nature of the simplex algorithm, it will often take a relatively large number of iterations to

prove infeasibility. Infeasibility reflects an overly constrained problem or mistaken assumptions. Or perhaps wishful thinking.

§ 6:3.3 Integer Programming Models

In real-world problems, it often happens that some of the variables in the linear programming model must have integer values only. For example, in optimizing an airline schedule, the number of flights per day between Chicago and New York can be seven or eight but not 7.4. Computer codes have been developed to solve problems in which certain variables can be either zero or one. Other integer values are generated using these zero/one or *binary* variables.

In leveraged leasing, zero/one variables are used to model changes in a basic rule. The tax law often specifies that rule one is to be followed up to a certain year or dollar amount, and then rule two applies. Zero/one variables can also model alternative constraints, in which either constraint one or constraint two must be met.

The presence of zero/one variables can greatly increase the amount of time required to solve the model. The computer solution proceeds by solving two linear models for each zero/one variable, one for the zero case and one for the one case. One zero/one variable will double the time to solution, two will multiply it by four, three by eight, etc. A dozen zero/one variables will multiply it by 4096. Actually, it isn't that bad, since the computer can avoid calculating cases that are inferior to solutions already found, but the bottom line is that integer models are much more time-consuming to solve.

Integer models are tricky to write, so we give a few examples. Suppose that all rents after the EBO date must be either all advance rents or all arrears rents. The linear model must include both advance and arrears rents after the EBO date so they will be available as a result. But left to itself, the model could (and probably would) choose a mixture of advance and arrears rent. We use an integer variable to make rents all advance or all arrears as follows:

```
For date    =  All rent dates after the EBO date:
   AdvanceRentalFactor(date)  <  z
   ArrearsRentalFactor(date)  <  (1 – z)
End of loop
```

Note that if the zero/one variable "z" is zero, the advance rental factors must be less than or equal to zero. Since rents must also be zero or greater, they are forced to be zero. If z is one, then (1 – z) is zero, and the same applies to the arrears rent. So the presence of z selects either advance or arrears rent only. We have assumed that rental factors could never really be as big as 100%. Often integer models require a "ridiculously large" number, and if 1.0 (usually representing the equipment cost) is not large enough, use M times z where M is a large number. Although M must be large, using too large a value may give rise to numerical instability. Usually it is apparent from the problem how large a value is needed.

As an example of either / or constraints, suppose that either x must be equal to 5%, or else y must be equal to 8%.

$$
\begin{aligned}
x &< 0.05 + z \\
x &> 0.05 - z \\
y &< 0.08 + (1 - z) \\
y &> 0.08 - (1 - z)
\end{aligned}
$$

If z is zero, then x must be less than or equal to 0.05, and greater than or equal to 0.05, so it must be equal to 0.05. Also, y must be less than 1.08 and greater than –0.92. We assume that this produces no real constraint on y. If it does, we need to multiply by a large "M" as discussed above. The opposite is true when z is one and (1 – z) is zero, so that y must be 8%. Note the frequent use of z for one case and (1 – z) for the other case.

Not all alternatives require an integer variable. Consider a fee based on a balance of some kind. The fee is 1% of the balance, except that if the balance is less than zero, the fee is zero.

$$
\begin{aligned}
\text{fee} &> 0 \\
\text{fee} &> \text{balance} \times 1\%
\end{aligned}
$$

Note that if the balance is positive, it controls the fee, but if the balance is negative, the fee will be zero. We have assumed that the fee will not exceed the minimum amount because it is an economic negative, and the model will try to minimize it. If this is not the case, we will divide the balance into a positive part and a negative part, and write the fee as a % of the positive part:

$$
\begin{aligned}
\text{pbal} - \text{nbal} &= \text{balance} \\
\text{pbal} &> 0 \\
\text{nbal} &> 0 \\
\text{pbal} &< 5z \\
\text{nbal} &< 5(1 - z) \\
\text{fee} &= \text{pbal} \times 1\%
\end{aligned}
$$

Now either pbal or nbal is zero thanks to the integer variable z. We multiplied by the large number 5 in case the balance really could be about as large as the equipment cost 1.0. If the balance varies with time (very likely) we will need a series of pbal, nbal and z, one for each period.

§ 6:4 Specialized Lease Structures

There are many enhancements and variations of the classic United States leveraged lease that have been used extensively by the leasing industry. They mainly grow out of changes to the IRS tax code regulations, which have been made throughout the years. We specifically discuss the current Section 467 regulations and how new structures have evolved based on them.

§ 6:4.1 467 Regulations

Leasing structures are very much dependent on the details in the official IRS code. On May 17, 1999, the IRS issued its final Section 467 regulations that relate specifically to leasing. This followed almost three years of uncertainty with the release of "proposed" final regulations. Provisions within these regulations effectively eliminated the profitable Lease-In/Lease-Out ("LILO") structure that had been used extensively in the late 1990s. At the same time certain safe harbor provisions were included with respect to rents that have permitted new structures to evolve.

According to the final Section 467 regulations all leasing transactions are now allowed an initial three-month rent holiday. No business reason is required to justify this interim period. If a valid business reason exists, then the rent holiday can be increased to the lesser of 24 months or 10 percent of the lease term. This more general rent holiday option can apply to any consecutive time period in the lease and need not occur at the beginning of the lease.

For real estate transactions, the 90-110 rental safe harbor has been expanded to 85-115. In addition, the regulations clarify how to calculate the average rent used in the safe harbor calculation for the uneven rent test in the event that the deal includes partial years. The numerator should be the total base term rent and the denominator should be the base term expressed in years. The calculation should be performed on a calendar year basis with the initial and last years treated as partial years.

The new regulations also clear up some previously unresolved issues. Transactions where the rent is adjusted to reflect a variable interest rate are allowed. A special safe harbor was created for different types of contingent rent. There is also a special transitional rule under which lessees may change their method of accounting to the constant rental accrual method for some agreements.

Under the new regulations, the 90-110 test must be applied over the entire term of all lessor options. One can no longer apply one 90-110 test through the EBO term and a second 90-110 test during the renewal period. This new rule substantially limits any benefit from the standard Pickle lease.

The new 467 regulations indicate that the lessor and lessee should recognize rental income in a consistent manner. This implies that the lessor should accrue forward his recognition of rental income from advance rent.

Of particular significance is the indicated treatment of prepaid and deferred rent. This allows for a separate treatment of cash rent payments and the stream of taxable rental income. These two streams of cash rent and allocated rent are now described independently, allowing for new structures that improve the lessee's benefit. In these new structures there is a decoupling of the rental income allocation from the rental cash profile.

Prepaid rent occurs when the cumulative cash rent received by a certain date exceeds that allocated for tax purposes. Deferred rent describes the situation when more rent has been allocated as taxable income than the actual cash rent received.

The key points are these:

1) The regulations define a safe harbor period of up to one fiscal year during which rental cash and allocated rental income can lag each other without penalty. This is equivalent to the cash rent being allocated to either the prior, current, or next calendar year. The allocated rent is used for the IRS Section 467 90-110 rent test and for taxable income. However, when an early termination occurs, the cash rent and allocations must be set equal, requiring a positive or negative correction to the rental cash, depending on whether the deal is in a deferred or prepaid state.

The first new structure, sometimes called the prepaid-deferred structure or allocated rent structure, allows independent cash rent and allocated rent streams that satisfy the 3-month rent holiday and the two-year safe harbor window and the 90-110 rent test provisions. The report § 6:9.7 illustrates the pattern of the annual cash and allocated rent streams for a sample lease with this structure.

2) If there is prepaid or deferred rent that does not fall within this effective two-year safe-harbor window, a 467 loan is defined, based on the cumulative difference between the cash rents and allocations. This loan accrues interest at a rate no lower than 110 percent of the applicable Federal rate and interest is adjusted at least annually on the deferred or prepaid amount. The interest is then recognized as taxable income or expense. This requirement has led to two additional structures.

The first structure applies when there is *adequate interest* on fixed rent when computing interest from the 467 loan. A section 467 rental agreement provides adequate interest on fixed rent if the above conditions on interest apply and if also, one of the following two conditions holds. Either there must be deferred but no prepaid rent and the sum of the present values of all amounts payable by the lessee as fixed rent (and interest, if any, thereon) is equal to or greater than the sum of the present values of the fixed rent allocated to each rental period. Or there is prepaid but no deferred rent and the sum of the present values of all amounts payable by the lessee as fixed rent, plus the sum of the negative present values of all amounts payable by the lessor as interest, if any, on prepaid fixed rent, is equal to or less than the sum of the present values of the fixed rent allocated to each rental period.

The applicable Federal rate (AFR) for a rental agreement means the Federal short-term rate if the term of the rental agreement is not more than 3 years, the Federal mid-term rate if the term of the rental agreement is more than 3 years but not over 9 years, or the Federal long-term rate if the term of the rental agreement is more than 9 years.

To compute a 467 loan, the principal balance equals the accumulated difference between accrued allocated rents and cash rents plus the interest, which has accrued on this difference. If this

balance is negative it will generate a negative interest amount in the next period, which will be considered as income to the lessor. Conversely, a positive balance will generate positive interest, which will be considered as an expense for the lessor.

The second structure applies when there is not adequate interest on fixed rent when computing interest from the 467 loan, as defined above. To compute the 467 loan, this structure uses proportional rental amounts, which are the amount of fixed rent allocated to the rental period multiplied by a fraction. The numerator of the fraction is the sum of the present values of the amounts payable under the terms of the section 467 rental agreement as fixed rent and interest. The denominator of the fraction is the sum of the present values of the fixed rent allocated to each rental period under the rental agreement. The only additional requirement is that the present value of the cash rents must equal the fraction multiplied by the present value of the allocated rents. In practice multiplying the proportionality fraction by a constant gives the same optimized rent streams, and so the formulation is reduced to the simpler problem using a fraction of 1.

With the disappearance of the LILO transaction, current deals have much lower after-tax yields and lessee EBO benefits. The upside to these new structures is that they are much less complex and require fewer lines of code and custom constraints to model them. There are also a few economic advantages to the new regulations over traditional leveraged leases. The three-month rent holiday can increase the lessee's EBO benefit from between 20 and 50 basis points. For example, assume we have a deal closing on October 2, 2007 with a 20-year base term, an 8% after-tax yield, and a 9.04% EBO benefit for the lessee. If we now add in a three-month rent holiday with the base term starting on January 2, 2008, the benefit increases to 9.49%. We can further increase the lessee's EBO benefit by 25 to 50 basis points if we allow the rental cash and income to lag each other by up to one fiscal year. Using our previous example with a three-month rent holiday, we go from a benefit of 9.49% to 9.79%.

Under normal circumstances, the adequate and proportional models do not add any economic benefit for the lessee. However, there are special situations when these models will produce superior results. One example is when the user is optimizing for the lowest average rent per year. The 467 loan models can accelerate the rental cash payments resulting in lower average yearly rents. An interesting observation in the adequate interest model is that the higher the 467 loan interest rate the better the economics become. However, it may be difficult to justify using an interest rate that is higher than the real debt rate for a particular deal.

§ 6:5 Search for Higher Yields

The leveraged leasing marketplace is extremely competitive. It is common for a lessee to receive many bids differing by only ten or twenty basis points on a big-ticket leveraged lease. The lessee wants the lowest present worth of rent. The lessor wants the largest economic return, which is often measured by the book yield in conjunction with other considerations (total earnings, quality of the equipment and the lessee). Still, each lease must operate within the constraints imposed by the tax law and IRS guidelines, credit considerations (rentals must repay the debt), and accounting. It is inevitable that the lessor will endeavor to obtain higher yields and lower lease

rates by inventing new structuring techniques and by reducing any slack in the various constraints.

The most likely source of greater economic returns is through a careful interpretation of the Code. If a lease structure can be found that increases the tax benefits in the lease, then a higher yield and/or a lower lease rate may be obtained. Unfortunately, the Code is often unclear on the exact tax treatment for a given lease structure. For example, the IRS guidelines are vague, and because they are guidelines for obtaining a ruling, and because case law provides weaker guidelines, a lessor may choose to ignore certain constraints. This problem is not particular to leveraged leasing, and the same tax uncertainty exists in other businesses. But it is especially important in leveraged leasing because even a small additional tax advantage can have a significant impact on yield and lease rates. When considering a leasing proposal, the lessor must determine whether a given lease deal has a tax posture that is more aggressive or more conservative than that adopted in the lessor's other businesses.

As we have seen, lessors can improve their yield by changing the leverage and the amounts of various debt services in the lease. It is standard procedure in leveraged leasing to use a computer to search for the most advantageous rent and debt structure. This technique is especially appealing because it does not involve a conflict with the Code or the lessee.

Another way to increase the apparent yield of a leveraged lease is to include recourse debt in order to "defer" the equity investment. That is, instead of making an equity investment at the closing, the lessor makes a small investment and signs a recourse note with a maturity of one or two years. The proceeds from this note help to pay for the equipment, and the repayment of the note is the final "installment of equity". A variation is to allow the debt service to exceed the rent. The excess of debt service over rent cannot be secured by the lease, and so the lessor becomes liable for it. This is also deferred equity.

The yield calculated for this structure will usually be higher than the yield calculated for the same lease without the deferred equity. However, the higher yield is an illusion. Although there is nothing wrong in principle with calculating the yield on a lease with a recourse loan included, comparing the yield with and without a recourse loan is like comparing apples and oranges. In one case, no financing of the investment is included even though the lessor may well borrow the equity investment. In the other case, partial financing of the equity investment is included explicitly in the calculation. The problem with a recourse note is more easily seen if several notes are used. It is possible to defer the equity investment until after a large amount of tax benefits are received. Then there is never any investment at all, the after-tax cash flow contains no initial negative and the yield is infinitely large.

The non-recourse debt is included in leveraged lease cash flow because the lessor is not obligated to repay it and because SFAS 13 specifies that the investment in a leveraged lease shall be recorded net of the non-recourse debt. Since the lessor is obligated to repay the recourse note, we suggest that the lessor analyze the lease without including the effect of any recourse financing. Alternatively, the lessor may analyze all leases including the effect of equity financing on a consistent basis.

If the lessor looks only at the yield, it may accept leases with equity financing and low lease rates, while rejecting substantially identical leases with higher lease rates and no equity financing. Of course, there is an exception to this rule if the recourse financing serves some useful purpose-for example, if financing were not available to the lessor from any other source.

The danger is that the equity investor will accept leases with recourse loans included and decline leases without recourse loans irrespective of other considerations such as lease rate. Or the lessor may mistake the higher yield with the book yield. Note that book yield must be measured without recourse loans. The Emerging Issues Task Force of the Financial Accounting Standards Board (the "Task Force") has addressed the issue of deferred equity. They have concluded that the present worth of the deferred equity must be added back to equity for purposes of calculating the book yield. Then the "interest" component (total deferred equity less the present worth) must be subtracted out of the book earnings to get the net book earnings. This "interest" subtraction occurs over the period of deferred equity, generally the first few years of the lease. When the Task Force first proposed this accounting treatment, some lessors happily noted that the revised yield could be higher than the old book yield (including the deferred equity cash flows) if the discount rate for deferred equity was high enough. Unfortunately, this is misleading. The "interest" subtraction lowers the earnings in the early years of the lease, and so the "real" book yield as measured by the earnings divided by the investment becomes variable. The "real" book yield is low during the deferred equity period and is higher later in the lease term. The "book yield" that a computer program finds in a deferred equity deal is just part of an intermediate calculation, and lessors must understand that yields on deferred equity deals are not comparable to other yields. The best procedure is simply to omit the deferred equity when analyzing the yield. When the Task Force first proposed this deferred-equity accounting, there was a marked decrease in deferred-equity deals. Deferred-equity deals had been on the increase due to highly competitive market conditions. It seems to be a fact of life that when yields are low, lessors would rather look at a higher number (deferred-equity yield) even though most know that it is invalid and not comparable to the real yield. The prevalence of deferred-equity deals is a good indicator of marketplace competitiveness.

In summary, we find that the highly competitive nature of the leveraged leasing marketplace has resulted in many novel lease-structuring devices. However, lessors must be wary of the assumptions upon which they base their yields. It will often be necessary to reverse some of these assumptions and to re-analyze the yield in order to compare different leases on a consistent basis. It will also be necessary to consider the aggressiveness of the tax assumptions.

§ 6:6 Return on Equity and Assets

Other yield methods based on accounting considerations are coming into more common use. The return on assets is a prominent measure of bank performance. Therefore, banks are concerned about the effect of a large number of leveraged leases on this ratio. The first step in the return on assets method is to calculate the book earnings from the lease. Then an allocation of overhead is made, and the interest expense of the investment funds is subtracted from book earnings. Interest expense arises from the assumption that part of the investment is made from borrowed funds. The interest expense is found from an accounting measure of the funds invested in the lease. The book earnings less overhead and interest is called net book earnings. The net book earnings are

then divided by the lease assets for each year (or quarter or month). This gives the return on assets for each year of the lease term. Because the return on assets varies, it is necessary to use an average that has been weighted by the dollar lease assets. This number is then used along with yield as a measure of lease value.

Another yield method, "return on equity," has become more popular in the last few years. There are two variations on this method. The first method is similar to the return on assets method. The book earnings are adjusted for overhead and interest expense. The result is then compared to the equity. The investment in the lease is considered to come from internal equity and debt. If the leasing company's leverage is 90%, and if the investment is $280,000, then the investment consists of $28,000 of "true" equity and $252,000 of debt. The book earnings less overhead and interest on the debt gives net book income. This net book income divided by the (small) equity gives the return on equity. The division may involve the present worth of earnings over present worth of equity, or an average may be used. Or an internal rate of return may be used.

The return on assets and return on equity methods described above are attempts to model a real leasing company, and to price deals to achieve specified accounting measures. Different lessors often have different incompatible ways of performing the analysis, depending on the assumptions they make about the financing of the leasing company, and the measure and allocation of overhead.

However, there is a second form of the return on equity calculation that has more general applicability. No allocation of overhead is made. The investment consists of equity and debt as before. The return on equity is a constant number throughout the lease term, and is found in a manner similar to the MISF yield method described above, except that the investment balance is considered to consist of debt and equity in a constant ratio. For each month of the lease term, the after-tax debt interest is the cost of funds multiplied by the debt balance in the previous month multiplied by one minus the tax rate (because interest expense is deductible). The return on equity is an after-tax return on equity rate multiplied by the equity in the previous month. The new investment balance is the old investment balance less the after-tax cash flow plus after-tax interest expense plus the return on equity "earnings". The new debt and equity are the investment balance multiplied by the fixed leverage ratio. The effect is to allocate the cash flow to the debt balance and the equity balance in proportion to the leverage, while calculating the debt expense after tax according to the interest rate and tax rate in effect at that time. The return on equity rate is a yield rate on equity, and for highly leveraged companies it will be several times the MISF yield rate. The return on equity for our example lease is 26.125% after tax. Note that this return on equity method is calculated on the after-tax cash flow, and is a variation on the regular MISF yield calculation.

The return on equity measures became more popular during the early 1980's when many industrial companies were entering the leveraged lease marketplace. These companies often evaluate investments using a hurdle rate of perhaps 20% for a return on their equity. The after-tax yields available were not at this level, so a return on equity measure was devised which was 20% and was more similar to what these non-financial companies were accustomed to seeing.

It is possible to derive the return on equity from a simple equation. If we write down the assumptions of the analysis as follows:

Capital = Equity + Debt

Debt = Capital X Leverage

Earnings on Capital = Yield X Capital

Debt Interest = Debt X Debt Rate X (1 - Tax Rate)

Return on Equity Earnings = Equity X Return on Equity Rate

Earnings on Capital = Debt Interest + Return on Equity Earnings

The above equations say that the capital invested is composed of equity and debt in a constant ratio, and that the return on equity is the return on capital less debt interest. The above equations can be solved to give:

$$\text{Return on Equity} = \frac{\text{Yield on Capital - (Debt Rate X Leverage)}}{\text{1 - Leverage}}$$

For our example, the yield on capital is the after-tax yield rate of 7.000%. The tax rate is 35%, and the after-tax debt rate is 7.5% times one minus the tax rate or 4.875%. The leverage is 90%, so the calculated return on equity is 26.13%. The point is that the return on equity method is so similar to the yield method that it does not contain any additional information. It simply re-states a perfectly valid yield on capital as an equivalent yield on equity, given a simple model of the capital structure of the lessor.

The question remains whether the return on equity is a meaningful number. We asked a number of prominent lessors this question in February 1987 and received a variety of answers. One said that the return on equity was the only meaningful number in a lease, because the leasing company's return in equity paid the bills. The return referred to was the return on equity of the leasing company, not the return as calculated above. Another lessor said that the return on equity was "mirrors" and was just used because it produces a larger number than the yield. Another lessor was thinking of using the return on equity as the principal measure of lease economics because it included the effect of variable tax rates. That lessor was also thinking of using the "true" before-tax yield. The brokers said that their job was to do the calculations requested by the investor in a prompt and accurate manner, and they often didn't have an opinion on the validity of the return on equity method.

We believe that the return on equity method of yield is not superior to the MISF yield, and that it is essentially equivalent to the MISF yield. It contains no new information. The lessor has two choices. Calculate the return on equity and compare it to the hurdle return on equity, but understand what the return on equity is. Do not think that the yield is "really" higher just because the analysis method has changed. Alternatively, calculate the required return on capital given the

hurdle return on equity and the known debt cost and leverage. Then compare the MISF yield with that yield on capital hurdle.

§ 6:7 Mathematics of Yield

Even though it is apparent that leases are usually chosen by reference to their accounting consequences, we turn to a mathematical discussion of yields. It is important to find a yield or other measure of lease value that will result in the selection of the best leases in the marketplace at the time. In addition, lessors must consider the complicated questions of how leasing compares with lending, how much money should be devoted to each, and how a portfolio of leases and loans should be chosen.

There is an enormous amount of theoretical literature on these subjects. Most of it speaks of the selection of investment projects by manufacturing firms, and the problem is called the "capital budgeting problem". For example, there are methods (present worth, yield analysis) available that are used to decide whether a firm should invest in a project such as a paper mill. The after-tax cash flow from the project is calculated given the available depreciation, the amount of the loans (if any), and the expected revenue and expense of the project. The literature then addressed how this after-tax cash flow should be evaluated, and whether the project should be accepted or rejected.

The cash flow from a typical project is similar to the cash flow from a leveraged lease, since the same elements are involved, namely depreciation and possibly loans. However, the analysis of a project must assume a certain level of project revenue, whereas the revenue from a lease is equal to the rents, which are fixed by contract. Since rents are much more certain than project revenues, we conclude that leveraged leases are more suitable candidates for this type of analysis than capital budget projects. Leveraged leases (and other financial transactions such as loans) are the ultimate in "projects".

The capital budgeting problem also considers the selection of a group of projects, or in our context, the selection of a portfolio of leases and loans. This analysis has culminated in "modern portfolio theory" (MPT), which is an attempt to choose the optimum portfolio of investments. MPT is currently the rage among pension fund managers, and at least two large computer programs are available for implementing it.

We turn, therefore, to a brief sketch of the literature, and include a few basic references for the interested reader. We will present the literature in the context of leveraged leasing, even though it is far more general and purports to analyze any conceivable project, whether it is an investment or a financing. It is directly applicable to the problems of lessees, such as lease-buy analysis and the evaluation of lease bids.

The work of Teichroew, Robichek, and Montalbano[1] addresses the decision whether to accept or reject a lease investment under the assumption that there is no uncertainty in the cash flows (no

[1] Teichroew, Robichek and Montalbano, Mathematical Analysis of Rates of Return under Certainty, 11 MANAGEMENT SCIENCE 395-403 (1965). Teichroew, Robichek, and Montalbano, An Analysis of Criteria for Investment and Financing Decisions under Certainty, 12 MANAGEMENT SCIENCE 151-179 (1965).

credit risk, no tax rate changes, etc.) and that the investor has an unlimited budget for lease investments. It assumes that the lessor has a known cost of capital, that the lessor can invest money in its business at a yield rate equal to this cost of capital. Alternatively, the lessor can invest money in a leveraged lease.

Teichroew et al. conclude that the lessor should invest in a leverage lease only when the yield measured using the MISF method is greater than the cost of capital. Since lessors evaluate the yield at a lower sinking fund rate, they are applying this criterion in a more conservative fashion. Teichroew et al. then demonstrate that this acceptable criterion is equivalent to investing in a leveraged lease if and only if the present worth of the cash flows (using a discount rate equal to the cost of capital) is positive. Note that present worth is the criterion for investing in a leveraged lease.

What Teichroew et al. have done is to provide a reason for the occurrence of multiple internal rates and to generalize the internal rate of return method to solve the problem. The leasing industry called their generalized internal rate of return method the multiple investment sinking fund method.

There is no implication that the MISF yield is a "true" yield, that this yield is "like an interest rate" or that leveraged leases with higher yields are preferable to leveraged leases with lower yields. This follows from the assumption that the lessor has unlimited funds to invest in leveraged leases and results in the conclusion that all leveraged leases should be accepted, provided that their yield exceeds the cost of capital. There is never any reason to choose among leveraged leases, and never any reason to rank them in value.

In the real world there are budget and other constraints, and it becomes necessary to rank lease investments in order of value, so that the most valuable leveraged leases may be chosen. This type of problem was first studied by Lorie and Savage[2]. They pointed out that while leveraged leases could be evaluated by measuring the internal rate of return or by measuring the present worth of the cash flows, these two methods often produce different rankings. That is, it is possible to find two leveraged leases such that the first lease has a higher internal rate of return and the second has a higher present worth. Which lease is more valuable?

Lorie and Savage showed that the difficulty in ranking leases in terms of value results from different assumptions as to the reinvestment of cash flows from the leveraged leases. With a single consistent choice of reinvestment assumption, both present value and internal rate of return analysis rank projects in the same order. It is usually assumed that cash flows can be reinvested at a yield equal to the cost of capital. Since this assumption is implicit in present worth analysis, present worth is the preferred method of ranking lease in order of value. Yield analysis does not have a consistent reinvestment assumption, since the reinvestment rate depends upon the yield rate.

We are therefore led to the conclusion that leveraged leases investments should be chosen to maximize the present worth of the portfolio, and that the present worth correctly ranks leases

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[2] Lorie and Savage, *Three Problems in Capital Rationing*, 28 J. BUS. 4 (1955).

according to value while yield analysis does not. The general statement and solution to the Lorie-Savage problem was provided by Weingartner[3], who suggests maximizing the present worth of the portfolio subject to the relevant constraints, such as the budget constraint.

The preeminence of present worth as a measure of leveraged lease value is based upon the choice of other lease investments. If we include the effects of risk and uncertainty in the analysis, the complexity increases drastically. Basic research into these problems has been provided by Weingartner, Bernhard[4], and Markowitz[5]. A general discussion can be found in texts such as those by Bussey[6] and Mao[7].

In summary, a large amount of literature concerning the capital budgeting problem is directly applicable to leveraged leasing, even though much of it is devoted to capital budgeting in manufacturing firms. Many of the conclusions are in flat contradiction to the thought and practice of many lessors. For example, the literature denies that cash flows with higher yields are necessarily more valuable than cash flows with lower yields even if all the cash flows are absolutely certain, whereas most lessors believe that higher yields are better. Also, the literature recommends present worth as a measure of lease value, not yield.

Most of the discussions of yield methods are devoted to expanding and circumventing the shortcomings of the internal rate of return method so as to bring it into conformity with results obtained from the present worth method. This contradicts the preference of lessors for a yield analysis rather than a present worth analysis. The literature states that, given a certain budget for leveraged leases, choosing leases with the highest yield does not necessarily produce the best portfolio. Choosing the highest present worth is preferable, but under certain circumstances, this does not produce the best portfolio either. This contradicts the lessor's view that choosing leases of higher yield will result in the best portfolio, given a certain quality of equipment and lessee.

A wealth of mathematics bears on the problems of lessors, lessees, and financial firms generally. It is an interesting area of research; one hopes that lessors will avail themselves of this recourse and mathematicians will devote their time to producing results that are of direct use to lessors.

§ 6:7.1 The Meaning of Yield

A lessor usually does not have the option of collecting a year's worth of leveraged leases and then choosing a portfolio of them all at once. The lessor must have a measure of the value of a lease that can be used on each leveraged lease as leases are presented to it. Also, most lessors do not feel comfortable selecting leases on the basis of present worth. They believe that a leveraged lease has a yield despite the admonitions of the mathematical finance literature. For these reasons, and for accounting reasons, most lessors base their investment decision on the MISF

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[3] H. Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems* (1963, 1967).

[4] Bernhard, Discount Methods for Expenditure Evaluation - A Clarification of Their Assumptions, 28 J. INDUS. ENG. 19-27 (1962).

[5] H. Markowitz, Portfolio Selection: Efficient Diversification Of Investments (1959).

[6] L. Bussey, The Economic Analysis of Industrial Projects (1978).

[7] J. Mao, *Quantitative Analysis of Financial Decisions* (1969).

Yield at zero sinking rate and the total book earnings, together with other considerations. In this section we explore this yield and the consequences of its use as a leveraged lease selection criterion.

First, consider the sinking fund and the use of a zero sinking fund rate. We have seen that many leveraged leases have unequal debt service amounts that may reduce the sinking funds to a very small amount and increase the yield. In an extreme case, it is possible to produce a leveraged lease with only positive cash flows after the investment. Therefore, the principal effect of the use of a zero sinking fund rate in leveraged lease marketplace has been to cause leveraged leases to be structured to avoid the creation of a sinking fund. That means that leveraged leases are structured as "pure investments" and function mainly as a use of funds (an investment) rather than a source of funds.

Financially speaking, lessors have borrowed less money than they could have and have paid off the loan faster than they need have, in order to avoid the situation in which they must receive large sums of money that must be paid back in the form of taxes. On the one hand, the lessor has been led to forego a valuable fund of cash in favor of repaying the non-recourse debt that the lessor is not personally obligated to repay. On the other hand, what is this fund of cash really worth?

Repaying the loan will result in an interest saving that is known. Not repaying the loan and generating a large sinking fund will result in an unknown amount of interest income (or interest saving on bank lines). Note that the sinking fund represents financing that appears as the liability "deferred taxes" rather than as the liability "loans" on the balance sheet.

A lessor interested in leveraged leases primarily as a source of funds (the sinking fund) rather than as a use of funds can use a yield method that fixes the yield rate and finds the sinking fund rate. This rate is interpreted as the interest rate at which the leveraged lease provides funds after deducting the cost of making the initial investment. Few lessors do this, because the funds are provided at the future time that is often beyond the planning horizon of the lessor, and because it is necessary to make an investment in order to obtain the loan (sinking fund). Nevertheless, the opportunity and financial benefits are present. Note also that a present worth evaluation of a leveraged lease will give a large value to the sinking fund. In general, leveraged leases would be structured quite differently if present worth analysis were used as the measure of lease value.

We must temper the foregoing conclusion by noting that older leveraged leases (often leases with low lease rates in which the lessor retains the ITC) still have a significant amount of sinking fund. These leases are both for use of funds (an investment) and as a source of funds. When a zero sinking fund rate is used, the value of the source of funds is neglected. This can be justified on the basis of conservatism since the value of the funds is currently unknown. However, a zero rate does seem too low.

We next turn to a discussion of the investment aspect of leveraged leasing. As we have seen, there are many leveraged leases that are close to pure investments. Most of the rest are viewed as pure investments by the investor. All single investor leases are pure investments because no significant sinking fund develops.

The principal feature of the investment in a leveraged lease is its short life. Tax savings are large and result in a rapid payback of the initial investment. Our example lease has after-tax earnings of $76,250 and a yield of 7.000% per year after-tax on an investment of $205,000. Multiplying 7.000% by $205,000 we obtain earnings of $14,350 for one year. The average life is $76,250 divided by $14,350, or 5.314 years. We conclude that a leveraged lease has a yield for only a short amount of time, and, therefore, even a fifteen-year lease is to be viewed as a short-term investment. A single investor lease has a much longer average life and stands between leveraged leases and loans from an investment point of view.

Note that even though leveraged leases have a relatively small amount of earnings they can have a large yield because the investment is outstanding for a small amount of time. An extreme case would be a twenty-year lease with zero assumed residual value, in which the investment was repaid with earnings in one year and all subsequent positive cash flow was used to pay taxes. Then the yield could be very high even if the earnings were small. However, the value of the leveraged lease would be low. Why would anyone invest money in a complicated deal and receive it back in one year? Note that the credit risk, tax-rate-change risk and other risks continue for the full twenty years, and that the yield calculation has ignored this. The yield calculation only measures the yield on the investment while there is an investment. If the investment exists only for a very short time, the yield becomes irrelevant.

However, many lessors demand a certain level of earnings as a condition to investing in a leveraged lease. This is equivalent to specifying a minimum average life. Combined with a minimum yield requirement, this is equivalent to the criterion that the yield must be adequate but also meaningful.

Of course, although the yield exists in this extreme example only for one year, the investment is returned in one year also. Therefore, it is possible to invest in another leveraged lease after one year. If another leveraged lease identical to the first is available after the first year, the lessor may invest in it and obtain the high yield for two years. Of course, the risk is twice as great, and this has again been ignored by the yield calculation. If another leveraged lease at the original yield were not available, the lessor would have done better to invest in a leveraged lease at a lower yield but with a two-year life. Note the similarity to deciding whether to lend money for one year or two years; the decision depends upon yield rates at the end of the first year.

In summary, a portfolio of leveraged leases is similar to a portfolio of short-term debt instruments with a major exception: the risk from a leveraged lease is reduced to zero at the end of the lease term, not when the investment is repaid. Therefore, any introduction of risk considerations can be expected to change the yield on a portfolio of leveraged leases much more than the yield on a portfolio of loans.

Although the foregoing discussion brings out an important point about leveraged (and other) leases, we must reconsider our conclusions when a secondary investment occurs. Most leveraged leases have an ending positive cash flow due to the receipt of the residual value. This results in another investment period just prior to the end of the lease term. Our conclusion that the average life of a leveraged lease is short remains true, but it is significant that the lessor is committing itself to two investments at the start of the lease term. We have discussed the first investment

above. The second investment must be made (due to tax payments) years later, often well past the lessor's planning horizon. Use of the MISF method ensures that the second investment will have the same yield as the first investment, but market conditions at the time of the second investment are unknown.

For some lessors, the principal motivation to invest in a leveraged lease is the residual value. A lessor may manage its portfolio to obtain the most residual value at the least cost, and ignore yield consideration. Since most leveraged leases are bid on and booked with an assumed residual value, the lessor must obtain a residual of at least this value in order not to suffer a reversal of earnings. But inflation may substantially increase the realized dollar value of certain assets. This makes the economic analysis one of estimating realized residual value rather than analyzing yields. Leveraged leases of real estate are in this category. However, it is still necessary to monitor risk and to budget for the large lease cash flows and tax liabilities.

In summary, we have found it convenient to discuss leveraged leases as a pure source of funds, as a pure short-term investment, as two short-term investments, and as a complicated way to buy a residual value. Single investor leases are always pure investments plus a residual value. For many leases, one of these elements dominates the investment decision, and economic analysis is relatively simple. However, many other leases contain several elements. It is not possible to compare different elements of a lease on a consistent basis unless a way is found to assign standard values to each element. This is just another example that "you can't add apples and oranges." Lessors evaluate the different elements by "feel" and reach an investment decision.

The process of valuing diverse leveraged lease elements must be specific to the investor and to assumed future market conditions. The various yield and analysis methods make use of simple models of the investor. The MISF method assumes that the investor has either an investment at the yield rate or a sinking fund at a sinking fund rate. The yield is then found by assuming a rate at which to value the sinking fund. The present worth method values both the investment and sinking fund at the same rate, and brings the result to the present as a dollar amount. The future worth method is similar. More elaborate models of the investor might be useful. In particular, a model including a simple adjustment for risk would improve the lessor's ability to choose leveraged lease investments, because it would make the yield more like the yield (interest rate) on a loan. Of course, a leveraged lease will never be a loan, and a sense of "feel" will always be needed in order to properly make a leveraged lease investment.

§ 6:8 Lessee Economics

The principal reason a company will acquire property by lease rather than purchase is an inability to use the tax benefits of ownership on a timely basis. Thus, the primary objective in a leasing transaction is often the sale of tax benefits to the lessor in exchange for a lease rate that is lower than the lessee's borrowing rate. If the lessee is subject to the Alternative Minimum Tax and the lessor is not, then a lease will result in a further tax benefit to the lessee. Usually, although not always, the cost of leasing to the lessee is greater than the cost of direct ownership if the lessee can use the tax benefits as effectively as the lessor, if the lessee's incremental tax rate is equal to or greater than the lessor's and if the asset has significant value to the lessee at the end of the lease term.

As we have seen, the analysis of the economics of a leveraged lease for a lessor must be made on an after-tax basis in order to properly evaluate the value of the deal. The situation is somewhat different on the lessee side. Most lessees enter into a leasing arrangement, as opposed to a direct purchase of the equipment, because they are not in a position to take advantage of the tax benefits of ownership; they expect to be in a tax loss position for a substantial portion of the lease term. In such a situation, a before-tax analysis will suffice to value the deal.

If a lessee anticipates turning taxable in the relatively near term, however, the picture changes substantially. Rent payments made by the lessee during a year in which it is taxable will represent a lower overall cost to the lessee than payments made during tax loss years. When the present value of the lessee cost is calculated on an after-tax basis, a deal which was optimally structured on a before-tax basis may suddenly appear sub optimal by a significant amount. A prudent lessee will consider the "real" benefit of a leveraged lease on an after-tax basis.

There are other reasons for leasing an asset rather than owning it. If the lessee knows that it will not need the asset after the lease term, it may seek a rent rate that includes a discount for the anticipated value of the residual. How much discount is available will depend on the lease term, and on the availability of lease funds at the time. This amounts to a sale of the residual value to the lessor rather than (or combined with) the sale of tax benefits. As a further reason for leasing, the lessee may find that greater financial leverage is available to it through a combination of leasing and traditional financing.

This last point is especially important for a lessee whose main problem is the availability of sufficient capital for its needs. The lessee may be able to obtain more funds through debt and equity sales in addition to leveraged leasing than it could through debt and equity sales alone. Because buyers of leveraged lease equity would not usually be investors in the lessee's securities, the lessee is going to a different market for funds. The extent to which additional funds may be acquired through leveraged leasing depends on the capital markets at the time.

Other reasons for leveraged leasing may arise from institutional or contractual constraints. For example, the lessee may have indenture restrictions against additional borrowing, but not corresponding restrictions against leasing. Or a manager may be able to acquire property through lease because a lease is an item of the expense budget, whereas a purchase would need to be approved as an item of the capital budget. Regulated industries may have differences in the accounting treatment of leasing versus ownership for rate-making or rate-base purposes, which may give an advantage to one or the other.

A leveraged lease can play a special role in a project financing or joint venture, since it can reduce the dollar size of the fixed financial obligation that must be provided through various purchase or supply contracts with third parties. This may make it possible to rely exclusively on third-party credit to finance the project.

The FASB in its Statement 13 has established accounting standards that would require the capitalization of some leveraged leases. That is, some leveraged leases would be recorded as assets or liabilities on the lessee's balance sheet, and the asset would be depreciated as if it were owned. Under the standards, leveraged leases must be capitalized if the equipment is leased for

75% or more it its estimated economic life, or if the present worth of the rentals is 90% or more of the equipment value less the ITC retained by the lessor (the "7d" test).

The discount rate specified in the standards is not necessarily the lease debt rate, but is either the lessee's secured debt rate for a loan with similar repayment terms or the lessor's implicit rate. The lessor's implicit rate is the internal rate of the rentals plus estimated residual value versus the equipment value less the ITC retained by the lessor. The lessee must use the secured debt rate, unless it is practicable to learn the implicit rate used by the lessor. In this case, the lessee must use the lessor's implicit rate if it is lower than the secured debt rate.

§ 6:8.1 The Lessee's Rent Cost Comparison

When lessees evaluate different leveraged leasing proposals, they usually evaluate the cost of the rents by either the present worth method or the internal rate method on a before-tax basis. Under the present worth method, the cost of the rents is given by the present worth of the rentals at the lessee's cost of capital. Under the internal rate method, the cost of the rents is given by the internal rate of the rentals versus the equipment cost. If the rents are level and equal in number, the two methods give the same cost comparison between two leveraged leases. That is, if lease A has a present worth cost lower than lease B, then it will have a lower internal rate also. This is simply equivalent to looking at one rent as the cost comparison.

If the rents are not level, it is possible (and likely) that if lease A has a present worth cost lower than lease B, then lease A may still have an internal rate higher than lease B. In this case, should the lessee choose lease A or lease B? The corresponding problem for the lessor was discussed above, and the mathematics of the "Capital Budgeting Problem" is completely applicable to the comparison of leveraged lease proposals and to lease-buy analysis. Lessees should evaluate rent costs by using the present-worth method, not the internal rate method.

In our example lease, the present worth of the rents is $803,953. If the EBO is exercised, the present worth of the rents up to the EBO date plus the EBO payments is $881,949. Both are at a discount rate of 7.5%.

§ 6:8.2 Lease-Buy Analysis

In evaluating the economics of a leveraged leasing transaction, the lessee should not combine the analysis of how to finance the equipment with the analysis of whether to acquire the equipment. Whether to lease or to own equipment, which the prospective lessee has already decided to acquire, is basically a financial decision. Having decided to acquire equipment, there are two common ways to evaluate the cost of financing, the "present-worth" method and the "basic-interest-rate" method. Neither has the relevance and market acceptance of the methods used to evaluate lessor economics.

The conventional method of comparing the costs of leasing and ownership is to discount the cash flows associated with the two alternatives to present worth at the lessee's opportunity rate. The present-worth cost of leasing is simply the present worth of the rentals plus the present worth of the residual value, all on an after-tax basis. The present worth of cost of ownership is the down

payment plus the present worth of the debt services less the present worth of the tax benefits (if they can be used). The tax benefits are the same as those that would be available to the lessor. Note that if the lessee is subject to the Alternative Minimum Tax, then taxes must be calculated for both the lease and buy cases using that method. The prospective lessee then chooses the alternative with the lower present-worth cost.

Although the present-worth method is the most common method, it is undoubtedly incorrect, unless the down payment amount is correctly chosen. The amount of down payment included in the analysis has a substantial effect on the apparent present-worth cost. The appropriate down payment amount should be associated with the prospective lessee's debt-to-equity ratio, rather than the actual down payment made on the equipment. In addition, the method has been criticized by Richard F. Vancil[8] on the grounds that it intermingles the effects of tax savings with the amount of funds provided. That is, if the prospective lessee always has alternative sources of financing available, the amount of financing obtained should not affect the results. In his view, the main difference between leasing and ownership is the difference in permissible income tax deductions.

Under Vancil's method of analysis, which he calls the "basic interest rate method", the cost of ownership is the price of the equipment less the present worth of the tax savings due to depreciation. This has the consequence that the cost of ownership is independent of the amount of the down payment. The cost of leasing is the price of the equipment less the present worth of the tax savings due to the deductibility of the principal portion of the rent. That is, the rents are divided into principal and interest payments as if they were debt services at a rate called the basic interest rate equal to the incremental borrowing cost. The principal portion is actually deductible, because it is really part of the rent, whereas it would not be deductible under ownership. The interest component of the rent would be deductible under either lease or ownership. This is the key feature of the lease according to this method of analysis. The method compares the total deductibility of rent versus the availability of depreciation. In this discussion we have ignored a problematic but crucial correction to the method (discussed in the reference by Vancil). It arises because the division of the rent into principal and interest payments leaves a balance other than zero at the end of the lease term unless the basic interest rate is equal to the lease implicit rate. The basic-interest-rate method is subject to criticism because it ignores the difference (if any) in the amount of financing provided and the effects of leasing and debt financing on the prospective lessee's capital structure. That is, it may not address the reasons the lessee may decide to lease in the first place.

In general, the quantitative analysis of the lease-buy decision has not reached the uniformity and market acceptance of the methods used to evaluate lessor economics. The findings of a study of lease-buy analysis methods were published[9] in 1980. This study attempted to find out the method of lease-buy analysis actually used by large firms, and to present a model based on a sound conceptual framework.

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[8] Vancil, *Lease or Borrow - New Method of Analysis*, 39 HARV. BUS, REV. 122-36 (1961).

[9] William L. Ferrara, James B. Thies, and Mark W. Dirsmith, *The Lease-Purchase Decision*, published by the National Association of Accountants and The Society of Management Accountants of Canada (1980).

In practice, lessees rarely go through the detailed quantitative analysis described above. Instead, they often decide to lease for some basic economic or other reason, such as the inability to use the tax benefits of ownership. They then attempt to obtain the best deal available by seeking lease rate bids, by using leveraged lease professionals, and by negotiation. Nevertheless, an important part of the decision to seek capital through equity or debt sales is the price comparison between the two. The lease equity market has its ups and downs as do other capital markets. If the lessee has the ability to compare confidently the prices of equity and debt with leveraged leasing, it will be able to obtain capital through leveraged leasing at the most advantageous times.

§ 6:9 Example Reports

§ 6:9.1 Taxable Income for the United States Leveraged Lease

Period Ends	Rent,Resid. & other Income	Depre-ciation	Interest on Loan	Amort. & other Expense	Taxable Income	Taxes Paid
12/2007	0	571429	15000	82	-586511	-205279
12/2008	83442	122449	54218	328	-93553	-32744
12/2009	83442	87464	52026	328	-56376	-19731
12/2010	83442	62474	49670	328	-29030	-10160
12/2011	83442	44624	47137	328	-8647	-3027
12/2012	83442	44624	44414	328	-5924	-2074
12/2013	83442	44624	41487	328	-2997	-1049
12/2014	83442	22312	38341	328	22461	7862
12/2015	92713	0	29096	328	63289	22151
12/2016	101984	0	23416	328	78239	27384
12/2017	101983	0	18609	328	83046	29066
12/2018	101983	0	18609	328	83046	29066
12/2019	101983	0	18609	328	83046	29066
12/2020	101983	0	12366	328	89289	31251
12/2021	101983	0	5383	328	96272	33695
12/2022	101983	0	0	328	101656	35579
12/2023	200000	0	0	0	200000	70000
Total	1590693	1000000	468385	5000	117308	41058

§ 6:9.2 Cash Flows for the United States Leveraged Lease

Period Ends	Taxes Paid	Rental Cash	Debt Service	Other Cash	B.T. Cash	A.T. Cash	Invest. + Fee	Net Cash Balance
12/2007	-205279	0	0	0	0	205279	205000	279
12/2008	-32744	27109	119200	0	-92091	-59347	0	-59068
12/2009	-19731	82346	82346	0	0	19731	0	-39337
12/2010	-10160	82264	82264	0	0	10160	0	-29176
12/2011	-3027	82176	82176	0	0	3027	0	-26150
12/2012	-2074	82081	82081	0	0	2074	0	-24076
12/2013	-1049	81979	81979	0	0	1049	0	-23027
12/2014	7862	81869	81869	0	0	-7862	0	-30889
12/2015	22151	156985	156985	0	0	-22151	0	-53040
12/2016	27384	101984	101984	0	0	-27384	0	-80424
12/2017	29066	85104	85104	0	0	-29066	0	-109490
12/2018	29066	118863	18609	0	100254	71188	0	-38302
12/2019	29066	101983	18609	0	83374	54308	0	16006
12/2020	31251	101983	98729	0	3254	-27997	0	-11991
12/2021	33695	101983	101983	0	0	-33695	0	-45686
12/2022	35579	98136	74468	0	23669	-11911	0	-57597
12/2023	70000	3847	0	200000	203847	133847	0	76250
Total	41058	1390693	1268385	200000	322308	281250	205000	

§ 6:9.3 Multiple Investment Sinking Fund Method - Monthly

Nominal Monthly After-Tax Yield .583333 % /month
Nominal Annual After-Tax Yield 6.999991 % /year
After-Tax Sinking Fund Rate .000000 % /year

Date	Investment	After tax Cash	Invest. Earnings	Payment of Investment	Ending Investment	Sinking Fund	S. Fund Earnings
10/2007	205000	0	0	0	205000	0	0
11/2007	0	0	1196	-1196	206196	0	0
12/2007	0	205279	1203	204076	2120	0	0
1/2008	0	-92091	12	-92103	94223	0	0
2/2008	0	0	550	-550	94772	0	0
3/2008	0	0	553	-553	95325	0	0
4/2008	0	8186	556	7630	87695	0	0
5/2008	0	0	512	-512	88207	0	0
6/2008	0	8186	515	7671	80536	0	0
7/2008	0	0	470	-470	81005	0	0
8/2008	0	0	473	-473	81478	0	0
9/2008	0	8186	475	7711	73767	0	0
10/2008	0	0	430	-430	74198	0	0
11/2008	0	0	433	-433	74631	0	0
12/2008	0	8186	435	7751	66880	0	0
1-2023	0	203847.15	776.25	133070.99	0	69999.91	0
2-2023	0	0.00	0	0	0	69999.91	0
3-2023	0	0.00	0	0	0	69999.91	0
4-2023	0	-17499.96	0	0	0	52499.95	0
5-2023	0	0.00	0	0	0	52499.95	0
6-2023	0	-17499.96	0	0	0	34999.99	0
7-2023	0	0.00	0	0	0	34999.99	0
8-2023	0	0.00	0	0	0	34999.99	0
9-2023	0	-17499.96	0	0	0	17500.03	0
10-2023	0	0.00	0	0	0	17500.03	0
11-2023	0	0.00	0	0	0	17500.03	0
12/2023	0	-17499.96	0	0	0	0	0
Total	205000	281250	76250	205000			0

§ 6:9.4 Multiple Investment Sinking Fund Method - Annual Totals

Nominal Monthly After-Tax Yield .583333 % /month
Nominal Annual After-Tax Yield 6.999991 % /year
After-Tax Sinking Fund Rate .000000 % /year

Date	Investment	After tax Cash	Invest. Earnings	Payment of Investment	Ending Investment	Sinking Fund	S. Fund Earnings
12/2007	205000	205279	2399	202880	2120	0	0
12/2008	0	-59347	5413	-64760	66880	0	0
12/2009	0	19731	4338	15394	51486	0	0
12/2010	0	10160	3466	6694	44792	0	0
12/2011	0	3027	3162	-135	44927	0	0
12/2012	0	2074	3196	-1122	46049	0	0
12/2013	0	1049	3302	-2253	48303	0	0
12/2014	0	-7862	3690	-11551	59854	0	0
12/2015	0	-22151	4885	-27036	86890	0	0
12/2016	0	-27384	6971	-34355	21245	0	0
12/2017	0	-29066	9497	-38563	59808	0	0
12/2018	0	71188	5661	65527	94281	0	0
12/2019	0	54308	2039	52269	42012	0	0
12/2020	0	-27997	3609	-31606	73618	0	0
12/2021	0	-33695	6170	-39866	13484	0	0
12/2022	0	-11911	7677	-19587	33071	0	0
12/2023	0	133847	776	133071	0	0	0
Total	205000	281250	76250	205000			0

§ 6:9.5 Rentals for the United States Leveraged Lease

Date	Arrears	Advance	Total	(% EC) Arrears	(% EC) Advance	(% EC) Total
10/01/2007	0	0	0	0.000%	0.000%	0.000%
01/01/2008	0	0	0	0.000%	0.000%	0.000%
07/01/2008	27109	0	27109	2.711%	0.000%	2.711%
01/01/2009	56333	0	56333	5.633%	0.000%	5.633%
07/01/2009	26013	0	26013	2.601%	0.000%	2.601%
01/01/2010	57429	0	57429	5.743%	0.000%	5.743%
07/01/2010	24835	0	24835	2.484%	0.000%	2.484%
01/01/2011	58607	0	58607	5.861%	0.000%	5.861%
07/01/2011	0	23569	23569	0.000%	2.357%	2.357%
01/01/2012	59874	0	59874	5.987%	0.000%	5.987%
07/01/2012	0	22207	22207	0.000%	2.221%	2.221%
01/01/2013	61235	0	61235	6.123%	0.000%	6.123%
07/01/2013	0	20744	20744	0.000%	2.074%	2.074%
01/01/2014	62698	0	62698	6.270%	0.000%	6.270%
07/01/2014	0	19170	19170	0.000%	1.917%	1.917%
01/01/2015	64272	78165	142437	6.427%	7.816%	14.244%
07/01/2015	14548	0	14548	1.455%	0.000%	1.455%
01/01/2016	0	90275	90275	0.000%	9.028%	9.028%
07/01/2016	11708	0	11708	1.171%	0.000%	1.171%
01/01/2017	0	75799	75799	0.000%	7.580%	7.580%
07/01/2017	9305	0	9305	0.930%	0.000%	0.930%
01/01/2018	16880	92679	109559	1.688%	9.268%	10.956%
07/01/2018	9305	0	9305	0.930%	0.000%	0.930%
01/01/2019	0	92679	92679	0.000%	9.268%	9.268%
07/01/2019	9305	0	9305	0.930%	0.000%	0.930%
01/01/2020	0	95800	95800	0.000%	9.580%	9.580%
07/01/2020	6183	0	6183	0.618%	0.000%	0.618%
01/01/2021	0	99292	99292	0.000%	9.929%	9.929%
07/01/2021	2692	0	2692	0.269%	0.000%	0.269%
01/01/2022	0	94290	94290	0.000%	9.429%	9.429%
07/01/2022	3846	0	3846	0.385%	0.000%	0.385%
01/01/2023	3847	0	3847	0.385%	0.000%	0.385%
Total	586023	804669	1390693	58.602%	80.467%	139.069%

§ 6:9.6 Termination Values

Date	Termination Value
10/1/2007	100.50000000%
01/1/2008	102.37092400%
07/1/2008	102.85621900%
01/1/2009	100.33942900%
07/1/2009	100.68918800%
01/1/2010	97.85138900%
07/1/2010	98.12517900%
01/1/2011	95.00142100%
07/1/2011	97.60172300%
01/1/2012	91.85748800%
07/1/2012	94.32324500%
01/1/2013	88.44735000%
07/1/2013	90.77384600%
01/1/2014	84.76195800%
07/1/2014	86.95154700%
01/1/2015	80.82985600%
07/1/2015	73.36225000%
01/1/2016	75.24391600%
07/1/2016	66.71858600%
01/1/2017	68.49048200%
07/1/2017	61.60311900%
01/1/2018	61.64873900%
07/1/2018	52.73754600%
01/1/2019	54.12340100%
07/1/2019	44.94431500%
01/1/2020	46.05278000%
07/1/2020	36.71401400%
01/1/2021	37.67468100%
07/1/2021	28.18045900%
01/1/2022	28.99973500%
07/1/2022	19.73351600%
01/1/2023	19.99998900%

§ 6:9.7 Prepaid/Deferred Rent, 1-Year Safe Harbor

Date	Cash Rent	Allocated Rental Income	Cumulative Cash Rent	Cumulative Allocated Rental Income	Cumulative Cash Rent Next Year	Cumulative Allocated Rental Income Next Year	Prepaid if Negative	Deferred if Negative
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)=(7)-(4)	(9)=(6)-(5)
12/2007	0.00	0.00	0.00	0.00	26,916.75	81,016.36	81,016.36	26,916.75
12/2008	26,916.75	81,016.36	26,916.75	81,016.36	81,016.43	162,032.72	135,115.97	0.07
12/2009	54,099.68	81,016.36	81,016.43	162,032.72	162,032.79	243,049.08	162,032.65	0.07
12/2010	81,016.36	81,016.36	162,032.79	243,049.08	243,049.14	324,065.44	162,032.65	0.06
12/2011	81,016.35	81,016.36	243,049.14	324,065.44	324,065.50	405,081.80	162,032.66	0.06
12/2012	81,016.36	81,016.36	324,065.50	405,081.80	405,081.86	486,098.16	162,032.66	0.06
12/2013	81,016.36	81,016.36	405,081.86	486,098.16	657,131.92	567,114.52	162,032.66	171,033.76
12/2014	252,050.06	81,016.36	657,131.92	567,114.52	756,150.56	657,132.02	0.10	189,036.04
12/2015	99,018.64	90,017.50	756,150.56	657,132.02	855,169.19	756,150.66	0.10	198,037.17
12/2016	99,018.63	99,018.64	855,169.19	756,150.66	949,503.83	855,169.30	0.11	193,353.17
12/2017	94,334.64	99,018.64	949,503.83	855,169.30	1,053,206.46	954,187.94	4,684.11	198,037.16
12/2018	103,702.63	99,018.64	1,053,206.46	954,187.94	1,152,225.10	1,053,206.58	0.12	198,037.16
12/2019	99,018.64	99,018.64	1,152,225.10	1,053,206.58	1,251,243.74	1,152,225.22	0.12	198,037.16
12/2020	99,018.64	99,018.64	1,251,243.74	1,152,225.22	1,338,722.99	1,251,243.82	0.08	186,497.77
12/2021	87,479.25	99,018.60	1,338,722.99	1,251,243.82	1,346,415.29	1,350,262.44	11,539.45	95,171.47
12/2022	7,692.30	99,018.62	1,346,415.29	1,350,262.44	1,350,262.44	1,350,262.44	3,847.15	0.00
12/2023	3,847.15	0.00	1,350,262.44	1,350,262.44	1,350,262.44	1,350,262.44	0.00	0.00
Total	1,350,262	1,350,262						